

Exam I, MTH 221, Spring 2011

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QUESTION 1. (39 POINTS) For each question below, circle the right answer.

(i) Let $A = \begin{bmatrix} -1 & 4 \\ -2 & 7 \end{bmatrix}$. Then A^{-1}

a. Does not exist

b. $\begin{bmatrix} 1 & -2 \\ 4 & -7 \end{bmatrix}$.

c. $\begin{bmatrix} 7 & 4 \\ -2 & -1 \end{bmatrix}$.

d. (a) and (b) and (c) are wrong



⇒

(ii) Let $A = \begin{bmatrix} -4 & 2 & 3 & b \\ 4 & -4 & d & a \\ 8 & -4 & -5 & c \\ 12 & -6 & -9 & -3b+2 \end{bmatrix}$. Then $\det(A)$

a. It depends on the values of a, b, c, d

b. it is always -90

c. it is always 16

d. it is always $-90(-3b+2)$



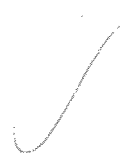
(iii) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -4 & 1 \end{bmatrix}$. Then $A^{-1} =$

a. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0.5 & -0.25 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix}$.

c. $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

d. None of the above



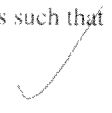
(iv) Let A, B be 3×3 matrices such that $2AB = I_3$ and $\det(A) = 2$. Then $\det(B^{-1}) =$

a. 16

b. 0.5

c. 4

d. None of the above.



(v) Let A, B as in the previous question. One of the following is a possibility for A^{-1}

a. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 0.5 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 4 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 4 & 2 \end{bmatrix}$

d. $\begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

⇒ (vi) Let A, B as in question (IV). Then $\det(2BA - 3I_3)$

a. -2

b. -8

c. -26

d. Can NOT BE DECIDED (i.e. More information is needed)

(vii) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}$, and $C = \begin{bmatrix} 2a_1 & 2a_2 + a_3 & 2a_3 \\ 2a_4 & 2a_5 + a_6 & 2a_6 \end{bmatrix}$. Given that B is a 3×3 matrix such that $AB = C$.

Then $B =$

a. $\begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

d. None of the above

(viii) Let A be a 3×3 matrix such that $A \xrightarrow{-2R_1 + R_3 \rightarrow R_3} A_1 \xrightarrow{3R_2 + R_3 \rightarrow R_3} A_2 \xrightarrow{2R_1} B =$

$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$. Then $\det(A) =$

a. -1

b. 4

c. 8

d. 2

⇒ (ix) Consider Question (VIII). Let F be a matrix such that $FB = A$. Then $F =$

a. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 3 & 1 \end{bmatrix}$

b. $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 3 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & -3 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 3 & 2 \end{bmatrix}$

No Answer ✓

$F = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix}$ should be ✓

(x) in Question (VIII). One of the following statements is correct:

a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A_1 = B$

b. $\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} A_1 = B$

c. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} A_1 = B$ ✓

d. NONE OF THE ABOVE

(xi) Given A is a 2×2 matrix such that $\left(A \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\right)^{-1} = 0.5I_2$. Then $A =$

a. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix}$ ✓

c. $\begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}$

d. NONE OF THE ABOVE

(xii) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ and $F = \begin{bmatrix} 2a_1 + a_3 \\ 2a_4 + a_6 \\ 2a_7 + a_9 \end{bmatrix}$. Given $\det(A) \neq 0$. Then the system of linear equations $AX = F$:

a. has infinitely many solutions and $x_1 = 2, x_2 = 0, x_3 = 1$ is a solution to the system.

b. has unique solution $x_1 = x_3 = 1$ and $x_2 = 0$

c. has infinitely solutions and $x_1 = x_3 = 1, x_2 = 0$ is a solution.

d. NONE OF THE ABOVE.

(xiii) Let $C = \begin{bmatrix} 0 & 1 & 4 \\ 1 & -1 & 3 \\ -1 & -1 & a \end{bmatrix}$. The system of linear equations $CX = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ is INCONSISTENT if

- a. $a = -11$
 b. $a = -4$
 c. $a = 4$
 d. $a = -7$

QUESTION 2. (11 POINTS) Solve the following system:

$$\begin{aligned} x_2 - x_3 + x_5 &= 0 \\ x_1 + x_3 - x_4 &= 2 \\ x_2 + x_3 + x_4 + x_5 &= 4. \end{aligned}$$

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$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 2 \\ 0 & -1 & 1 & 1 & 1 & 4 \end{array} \right] \quad R_1 + R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{array} \right] \quad R_3 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{array} \right]$$

Read: x_1, x_2, x_4 are leading variables
 x_3, x_5 are free variables

$$x_2 - x_3 + x_5 = 0$$

$$x_1 + x_3 + 2x_5 = 6$$

$$x_4 + 2x_5 = 4$$

one solution: $x_1 = 6$
 $x_4 = 4$
 $x_2 = x_3 = x_5 = 0$

initial solution:

$$x_1 = 6 - x_3 - 2x_5$$

$$x_2 = x_3 - x_5$$

$$x_4 = 4 - 2x_5$$